**Math Review**

**Learn Mathematical Induction**

**Example. Please follow this style and prove the result in Channel Intro and bring to the first day of the class.**

Prove by induction 1^2 + 2^2 + ... + n^2 = n(n+1)(2n+1)/6.

Proof.

Step 1. Prove base case(s).

Let n = 1.

Then LHS = 1^2 = 1 and RHS = 1(1+1)(2+1)/6 = 1.

Hence base case is proved.

Step 2. State induction hypothesis.

Assume the result is true for n = k,

1^2 + 2^2 + ... + k^2 = k(k+1)(2k+1)/6

Step 3. Induction step. **//This is the important step. Doing step 1 and 2 will get you 0 credit in the exam.**

Prove the result for n = k+1.

//At this point write down the LHS for n = k+1

LHS = 1^2 + 2^2 + ... + k^2 + (k+1)^2                                   **//25% credit**

//Use induction hypothesis

LHS = k(k+1)(2k+1)/6 + (k+1)^2                                              **//50% credit**

//Simplify LHS

k(k+1)(2k+1)/6 + (k+1)^2 = (k+1)[k(2k+1)/6 + (k+1)]

                                          = (k+1)[2k^2 + k + 6k + 6]/6              **//70% credit**

= (k+1)[2k^2 + 7k + 6]/6                    **//90% credit**

= (k+1)(k+2)(2k+3)/6 = RHS              **//100% credit**

**Note:**

Step 3. Induction step. **//This is the important step. Doing step 1 and 2 will get you 0 credit in the exam.**

Prove the result for n = k+1.

//At this point write down the LHS for n = k+1

LHS = 1^2 + 2^2 + ... + k^2 + (k+1)^2                                   **//25% credit**

//Use induction hypothesis

LHS = k(k+1)(2k+1)/6 + (k+1)^2                                              **//50% credit**

//Simplify LHS

k(k+1)(2k+1)/6 + (k+1)^2  =   RHS                                          **//You never proved the result, You will get 50%**

There are many videos on Youtube that will review college algebra. Here is one. Feel free to post your favorite.

<https://www.youtube.com/watch?v=ouUaxWVJNSI>

**Arithmetic and geometric series**

<https://www.youtube.com/watch?v=Uz4wjNsMKP8>

<https://www.youtube.com/watch?v=xavgv1m9feE>

**Aithmetico-geometric series**

<https://www.youtube.com/watch?v=jZxUWaLqzW0>

**Logarithms**

[**https://www.youtube.com/watch?v=Z5myJ8dg\_rM**](https://www.youtube.com/watch?v=Z5myJ8dg_rM)

**log a + log b != log (a + b)**

**log 1 + log 2 + log 3 + log 4 is not 4 \* (log 1 + log 4)**

log 1 + log 2 + log 3 + ...+ log n is not an arithmetic series.

So you can not use the formula   n \* (first + last)/2

**(k + 1)/(k + 2) != 1/2. You cannot cancel k.**

**Example 2 (induction)**

**Theorem. For all n ≥ 0, n5 – n is divisible by 10.**

**Proof.**

Base cases n = 0 and n = 1. (we need two base cases. You will see the reason. Keep reading!)

Clearly, the result holds.

Induction hypothesis.

Assume the result is true for all values of n in the interval [0, m].

n5 – n is divisible by 10 for 0 ≤ n ≤ m.

Induction step.

We need to prove the result for the next value. That is, we need to

prove that (m + 1)5 – (m + 1) is divisible by 10.

(m + 1)5 – (m + 1) = [(m – 1) + 2]5 – [(m – 1) + 2]

= (m – 1)5 + **10 (m – 1)4 + 40 (m – 1)3 + 80 (m – 1)2 + 80 (m – 1)** + **32**– (m - 1) – **2**

(everything in bold is a multiple of 10)

= (m – 1)5 – (m – 1) + **10 (polynomial in m).**

By induction hypothesis, (m – 1)5 – (m – 1) is dividable by 10. Hence the proof.

**(Since we are using m - 1 and not m, we need two base cases.)**

**Example 3 (induction)**

**Prove by induction n^2 < 2^n for n > 4.**

Proof.

Step 1. Prove the base case.

Let n = 5.

Then LHS = 5^2 = 25 < 32 = 2^5 = RHS.

Hence base case is proved.

Step 2. State induction hypothesis.

Assume the result is true for n = k > 4.

k^2 < 2^k for k > 4.

Step 3. Induction step.

Prove the result for n = k+1.

RHS = 2^(k + 1) = 2^k.2 = 2^k + 2^k > k^2 + k^2 (by induction hypothesis) .

Note that k^2 = k.k > 4k (since k > 4).

Hence RHS = 2^(k + 1) > k^2 + 4k = k^2 + 2k + 2k > k^2 + 2k + 1 (since 2k > 8 > 1).

Thus RHS = 2^(k + 1) > (k + 1)^2 = LHS